On Star Corrections. By Professor H. H. Turner, M.A., B.Sc.

1. The foregoing paper by Mr. Cooke affords an opportunity for putting down some notes on this subject, which has occupied my attention at intervals for some years past.

2. To the end of the year 1885 star corrections were computed at the Royal Observatory, Greenwich, by means of Airy's constants and day numbers, which were essentially those of Bessel, slight modifications being made so that algebraical signs were avoided. The day numbers were printed in the *Nautical Almanac* to the year 1890, but discarded when it was known that they were no longer required at Greenwich. The "star constants" were printed in the Greenwich Catalogues up to and including the Nine Year Catalogue 1872; and their computation was a serious addition to the labour of constructing the Catalogues.

3. There were several reasons for making a change, but the two chief were—

(*a*) The adopted procedure was found to be cumbersome.

(*b*) The use of the so-called "star constants" from year to year was found to be a fruitful source of errors, both because the numbers were not really "constants" and their slow change were apt to be neglected too long, and because errors were made in copying.

4. From 1886 January 1 Mr. Stone's "Tables for facilitating the computation of star constants" have been regularly used. All the computations are made independently in duplicate, and the results compared. A time about equivalent to the whole time of two computers is thus occupied in the computation of star corrections. If the meridian observing were in any way distributed in zones, this labour might of course be considerably reduced; but distributed, as it is at Greenwich, to all zenith distances indiscriminately, there is no way of reducing the labour except by simplifying the processes, if possible.

5. Mr. Finlay, in *Monthly Notices*, vol. l. p. 497, gives a method which certainly has advantages, and Mr. W. E. Cooke's method, given above, also seems to me to be a distinct simplification. My only doubt is whether the advantages gained in either case are quite sufficient, say, to make a change at Greenwich desirable. At the same time the question of saving labour is one of great importance, and the matter cannot be too thoroughly discussed. I have tried various methods during the last few years; in the first instance various forms of tabulation, which after many trials were found to be too cumbrous; and latterly different mechanical devices. At least four mechanical devices have been tried with fair success, as shown by models in wood and paper; but I do not propose to call attention here to more

D D

than one of these—the last and simplest—which I hope will prove efficient.

6. The star corrections in R.A. and Decl. $\Delta\alpha$ and $\Delta\delta$ may be written, as on p. 297 of the *Nautical Almanac* for 1894,

$$\Delta\alpha = f + g \sin(G+\alpha)\tan\delta + h\sin(H+\alpha)\sec\delta \quad . \quad . \quad . \quad . \quad (1)$$

$$\Delta\delta = i\cos\delta + g\cos(G+\alpha) + h\cos(H+\alpha)\sin\delta \quad . \quad . \quad . \quad . \quad (2)$$

and the first of these equations may be written

$$\Delta\alpha\cos\delta = f\cos\delta + g\sin(G+\alpha)\sin\delta + h\sin(H+\alpha) \quad . \quad . \quad . \quad (3)$$

the expression on the right being now similar to that of equation (2).

Now all the terms in equations (2) and (3) may be represented under the form

$$p\sin(P+\alpha)\sin\delta,$$

where $\sin\delta$ is in some cases unity, i.e. δ may be put permanently equal to 90°. Thus a mechanical device which will give such a term at sight by setting for α and δ (p and P being set at the beginning of the series) would save the labour of entering three logarithms, adding, and taking out antilogs. There is, of course, nothing new in the principle of such devices; it only remains to settle the details, and I propose the following arrangement:—

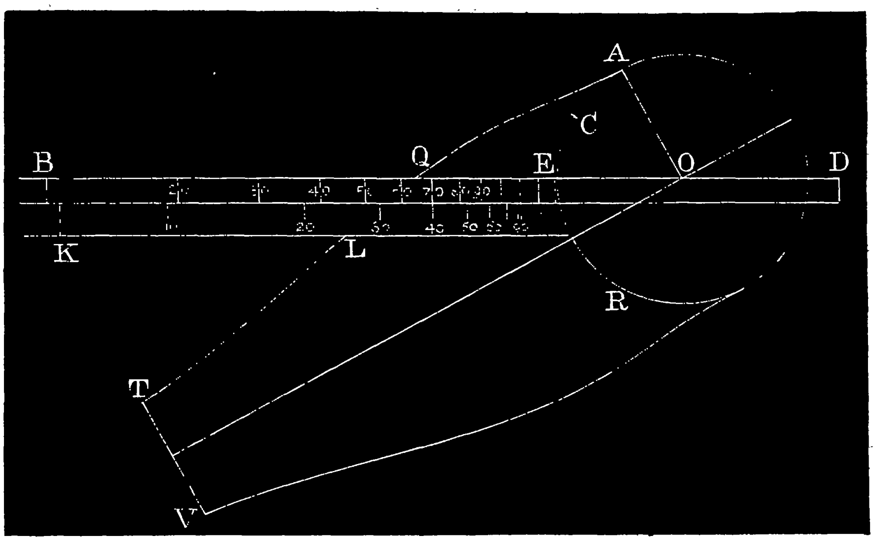

Fig. 1.

7. A graduated circle ACR can turn about O on a plate TAV. The point C is the zero, and R.A. is set off in the direction CR. The arc CA is set equal to the angle P (representing either G or H in the star corrections) and the circle then clamped to the plate. The plate and circle can now be turned about O until CE reads the R.A. of any particular star; and

thus the arc $AE = (P + \alpha)$. The plate VAT is so constructed that on any radius, OQ, the part EQ outside the circle is proportional to log cosec AE, or $-$ log sin AE. So that if on a scale BD, BE represent log p, then

$$BQ = BE - EQ = \log p - \log \operatorname{cosec} AE = \log p \sin (P + \alpha).$$

If, further, another scale KL slide along BQD, and be graduated proportionally to log sin δ from left to right; and if the point corresponding to 90°, where

$$\log \sin \delta = 0,$$

be brought vertically under Q, then if S be a point on the scale BD vertically over the point for declination δ on KL, we shall have

$$\begin{aligned} BS &= BQ - QS \\ &= \log p \sin (P + \alpha) \sin \delta. \end{aligned}$$

The operations thus consist of the following:—

(a) *Preliminary*, applying to the whole of a set: Turn the circle on the plate until $AC = P$, and clamp circle on plate. Slide BD until the point corresponding to p is on the limb of the circle and clamp BD. It would be well to have lenses or even microscopes, at least for these permanent settings.

(b) For any star α, δ: Bring α to E, the edge of BD; set the 90° mark on KR under Q, where the plate cuts BD, and read off the point S on BD vertically over δ. There should be no difficulty in doing this if BD be graduated on both edges.

8. This being the general principle several small modifications of detail suggest themselves. In the first place we have *two* terms involving G and g and two involving H and h. Consider the first. If we have a scale B′D′ precisely similar to BD but at right angles to it, and cutting the circle in E′, the plate in Q′ &c., then if

$$\begin{aligned} BQ &= g \sin (G + \alpha) \\ B'Q' &= g \cos (G + \alpha). \end{aligned}$$

Thus the two terms $g \sin (G + \alpha) \sin \delta$, required for $\Delta\alpha \cos \delta$, and $g \cos (G + \alpha)$, required for $\Delta\delta$, can be read off at one setting. Similarly writing $(H + 90°)$ for G, the other two terms can be found at one setting. No subsidiary scale KLR is necessary for the scale B′D′, since sin δ only occurs in one term of each pair.

Secondly, in all logarithmic scales there is a great inconvenience arising from the length to which they extend for the small quantities. The length for the portion 0·1 to 1·0 is as great as that from 10 to 100, or from 100 to 1,000, and is, of course, precisely similar. We can thus take advantage of this fact by using the same portion twice or three times over, always remembering to divide by 10,

D D 2

or 100, or 1,000, as the case may be. Thus the plate VAT is terminated abruptly at T, whereas it should really

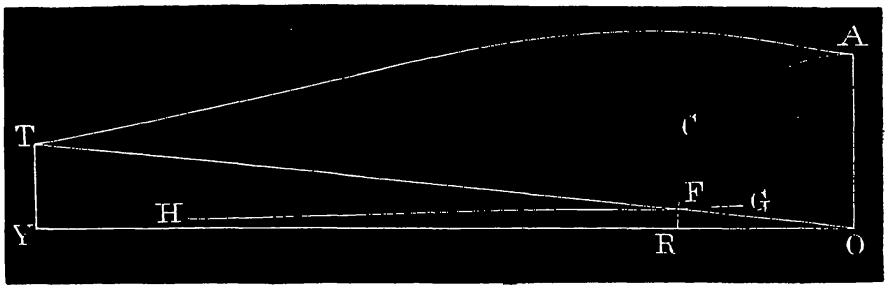

Fig. 2.

extend to infinity. The point T corresponds to the angle $5^\circ\ 44'$, whose sine is 0.1. If OT cut the circle in F, and GFH be drawn so that excess of radius vector is proportional to log. cosec (angle) $-$ 1, or log. $\left(\frac{1}{10}\text{ cosec angle}\right)$, instead of log. cosec (angle), then the curve GFH will read off on the scale

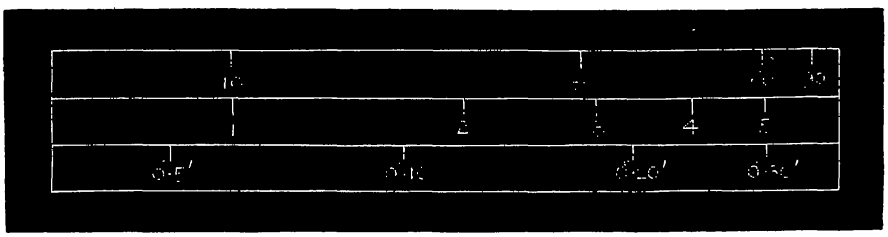

Fig. 3.

BQD *ten times* the corresponding term, and may be used for small angles. Similarly another curve may be drawn reading off *one hundred times* the term for smaller angles still.

Again, the scale for δ need not be of more than a moderate length; it need not, in fact, extend beyond $5^\circ\ 45'$ if below the portion $90^\circ - 5^\circ$ is placed another portion corresponding to log. (10 sin δ) or 1 + log. sin δ instead of sin δ, and below this another corresponding to log. (100 sin δ) or 2 + log. sin δ. The scale BD need thus not be extended beyond modest limits. I think convenience would be sufficiently consulted, and sufficient accuracy attained, by making the portion from log. 10 to log. 100 equal to one metre, though a smaller contrivance than this might answer.

9. The above was already in type when a series of simpler contrivances suggested themselves, which promise to be many ways more efficient. It will, I think, be of interest to describe four successive devices which are all comparable in efficiency, though the last will probably turn out the best. The

use of a logarithmic scale, with the inconveniences detailed in paragraph 8, is discarded in the following apparatus.

10. On a drawing board VQ (fig. 1) are drawn a series of "curves of sines" AC_1B, AC_2B, AC_3B, &c., the maxima OC_1, OC_2, OC_3, &c., being respectively 30, 29, 28 . . . . 2, 1, 0 units —say, half-inches—and representing 30″, 29″, 28″ . . . . 2″, 1″, 0″. The base AB being divided uniformly from 0^h to 12^h, as at DE,

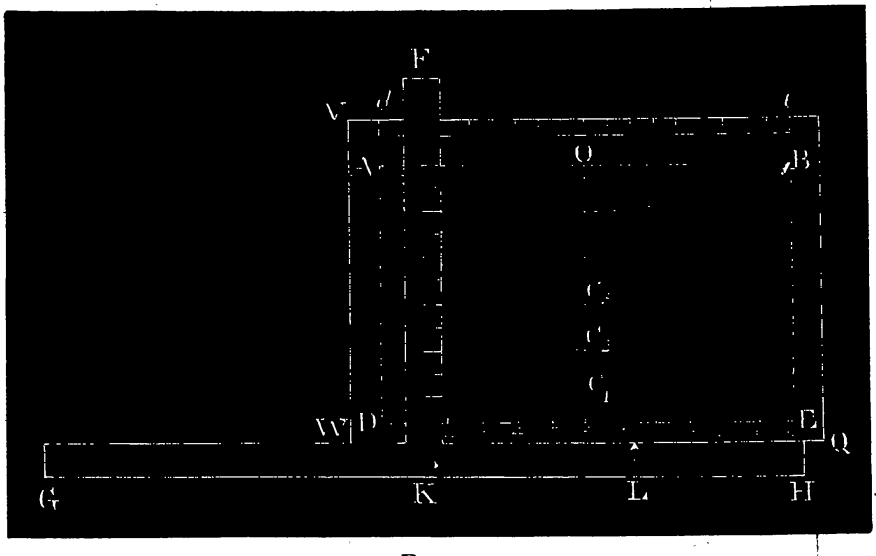

Fig. 4.

or from 0° to 180°, as at *de*, ordinates to any one of the curves, say, maximum 28″, give the values of 28″ sin a for the various R.A.'s or angles from 0^h to 12^h; and a T square FGKH whose T slides along the base WQ of the board, and whose edge FK is graduated uniformly, may be used to read off the value of this quantity by noting the point where the curve 28″ cuts its edge when slid to the proper R.A. If the value of 28″·3 sin a is required, we must interpolate between the curves for 28″ and 29″, either by eye or by actually drawing the 10 intermediate curves, which is not difficult on the scale suggested. Thus we see how to read off p sin a, where p has any value from 0″·0 to 30″·0. To read off p sin $(P+a)$ we merely alter the index on the T square from K to some other point L such that when L points to a, K points to $(P+a)$, or $LK = -P$. Thus L is constant in position for the day. An ink mark on a piece of paper gummed temporarily to GH would be an adequate index.

Now to form p sin $(P+a)$ sin δ we merely move the square to δ (taking now the top scale *de* divided in degrees) and read off the value of the curve whose maximum is p sin $(P+a)$, as already found.

When once the curves are nicely drawn this makes a very efficient machine. Its one drawback is the difficulty of following the curves, when these are numerous enough to give sufficient

accuracy. To obviate this the apparatus of the next paragraph was devised.

11. Instead of many curves, a single curve AC_1B would be sufficient if we could vary the scale on FK so as to make it read p units at the point C_1. Now a scale can be varied by the device used in the "diagonal scale." If the line OA (fig. 5) be divided

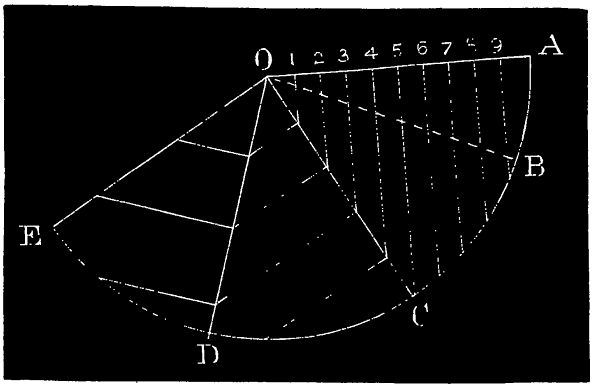

Fig. 5.

into ten equal parts, and perpendiculars be erected to OA at each point, then any other radius OB of the same length as OA will be divided uniformly into a smaller number of parts. For

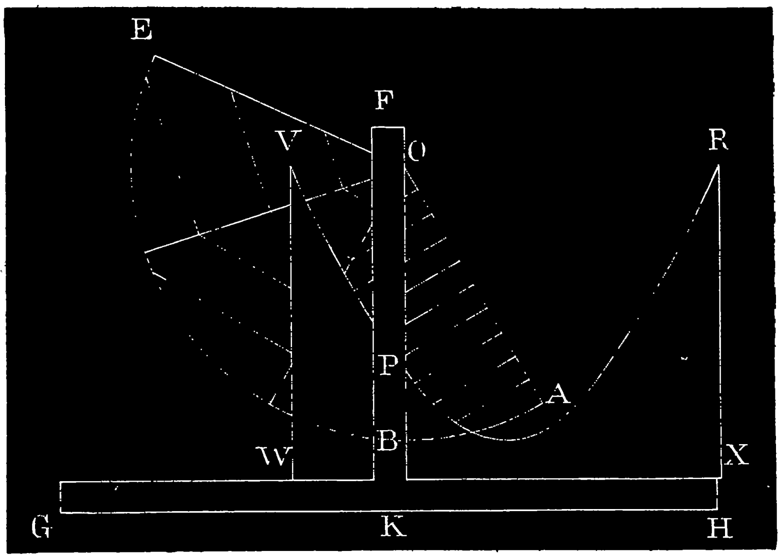

Fig. 6.

convenience new perpendiculars may be drawn at any other radius OC, OD, &c. Let us call a circular sector so divided a "reducing scale." Thus if FGH (fig. 6) be again our T square, and a reducing scale OAE be pivoted at O on the edge FK, then the maximum ordinate OB can be divided into any arbitrary number of parts p by rotating the reducing scale. Let us now cut out of card a single sine curve VAR,

which may be slid beneath the arm FK of the T square, the edge WX of the card bearing against the T of the square. Then the ordinate OP will be $p \sin(P+\alpha)$, as before, if the index be adjusted so that $(P+\alpha)$, and not α simply, be read off on the sliding scale.

12. So long, however, as we use a *slide* for setting in R.A. we get the sine curve entering into the machine. If the R.A. is to be set off on a uniform scale—which is practically a necessity if we are to form the sum $(P+\alpha)$ mechanically—we are limited to two methods—those of translation and of rotation. It is curious that I should have dwelt so long upon the *slide* for R.A., for the graduated circle is obviously more appropriate. But one seldom thinks of the simplest thing first, and it was not until I had made one or two machines of the above type that it struck me to try setting off the R.A., not on a straight line, but on a circular arc. The curve now to be used is no longer

$$y = a \sin x + b,$$

but

$$r = a + b \sin \theta,$$

where a and b are constants. It is obviously simplest to take $a = 0$, when the curve is a circle and can be described mechanically with great ease. The apparatus now becomes modified as

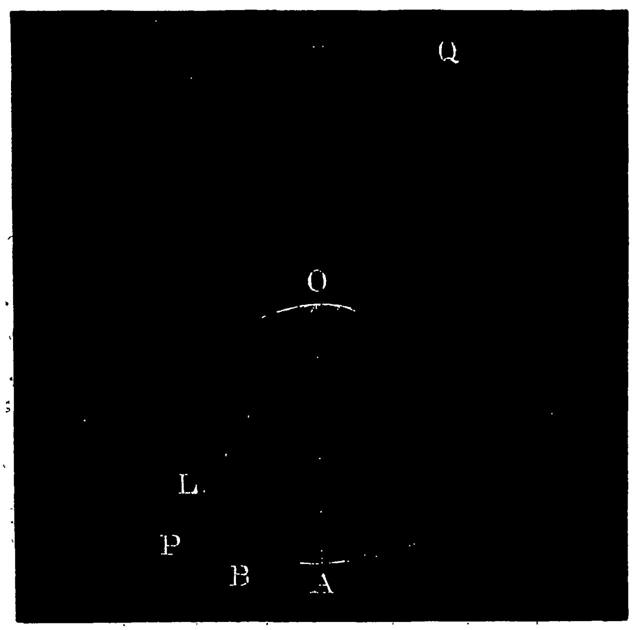

Fig. 7.

follows: The R.A. or declination is set off on the rim of a circle APQ, but so that the zero point can be shifted arbitrarily to any point B, where AB is made equal to P. A series of circles are described, all passing through O, and with diameters 30″, 29″, 28″, &c. . . . . 2″, 1″, corresponding to the series of sine curves in paragraph 10 (fig. 7). Then if BP $= \alpha$, so that AP $= P + \alpha$, it

is easily seen that OL=OA cos (P+a), and the value of p cos (P+a) can be readily found by tracing the circle corresponding to the value p up to the radius OP, which should be a divided scale.

13. But finally the circles need not be actually drawn, and herein consists the great simplicity of what I believe to be the final device. If OT (fig. 8) be the length p, and if OP be a straight-edge, and a set-square MLK be slid along OP so that the edge

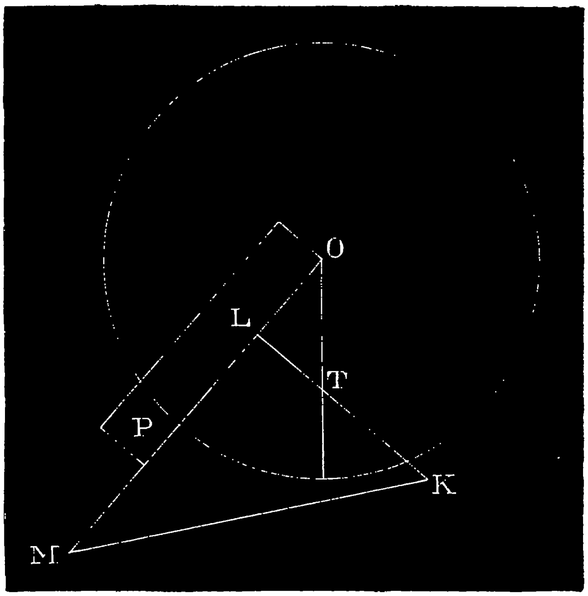

Fig. 8.

LK passes through T, then obviously OL=OT cos (P+a), LT=OT sin (P+a); in fact, we have arrived by a roundabout route at the very simplest method of finding the sine and cosine required by actually constructing the triangle of which they are the sides, viz. OLT. The final machine I suggest is, therefore, as follows (fig. 9). I describe it in terms of wood and paper, though a more exact apparatus can obviously be made in metal.

On a drawing board covered with white paper a circle is drawn, and carefully graduated. (It is convenient to have graduations extending from the point A in the direction AD in degrees, and in the direction AE in hours and minutes.) A straight-edge POP′ is pivoted at O on its edge (a piece of paper may be gummed on in order to make a hole for the pivot), and graduated uniformly from O outwards. A set-square MLK, graduated on one edge LK, completes the apparatus. (The edge LK should also be graduated *on the back*, so that the set-square may be reversed if necessary. The straight-edge may be reversed by simple rotation.)

To form the terms p sin (P+a) and p cos (P+a) first find the position of the point T for the day by turning the scale OP

to the point D, where AD=P, and setting off OT=p. Then, when OP is turned to any right ascension a, where AE=a, we have EOD=(P+a), and if the set-square be slid up to the point T the distances LT and OL, which can be at once read off the scales, are the quantities required. The point T may be marked

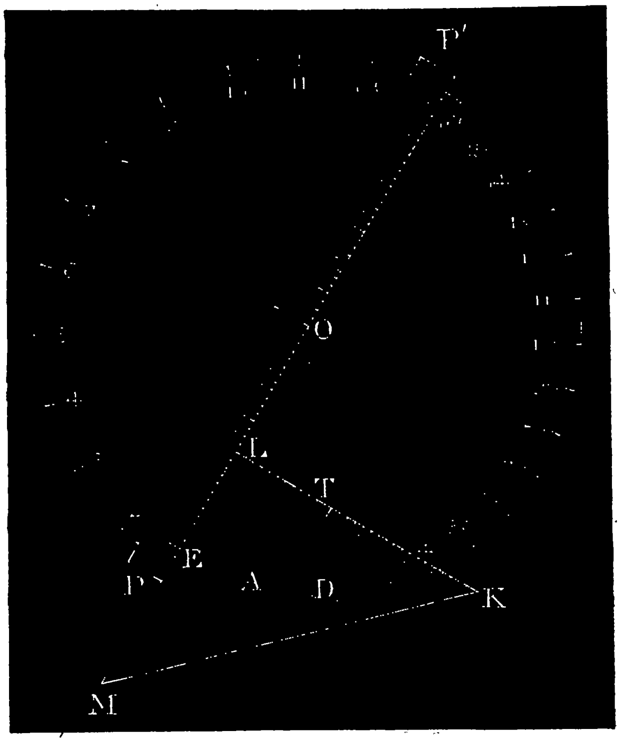

Fig 9.

for the day by an ink dot. The successive positions of these dots will form a curve which may be laid down on the paper for a whole year, and dates attached to the proper points.

14. This brief description of the machine will probably be sufficiently intelligible, and, as I have not yet had an opportunity of trying it thoroughly in practice, any practical comments are more fitly reserved for a future occasion. But I cannot but think that a simple apparatus of the kind will at least be found of great use in *checking* computations, though it may be advisable to make the computations originally in figures.

Graduating Wedges. By Captain W. de W. Abney, C.B., R.E., D.C.L., F.R.S.

In some cases where wedges are very opaque it becomes difficult to arrive visually at the correct graduation, as light from an ordinary source, such as a lamp, will not pass through with sufficient quantity to enable an accurate measure to be made. The light which it seems most desirable to employ is the electric (arc) light, since it is far more luminous than a lamp or candle, and it has the advantage that a large quantity of light is emitted from what is practically a point. There are, however, variations in the electric (arc) light from time to time, and unless the comparison light, with which the relative intensities passing through different parts of the wedge are measured, varies at exactly the same time and in the same proportion, the measurements will often be very much out. If we merely wish to measure the white light transmitted, the apparatus to employ is not very extensive, and fig. 1 will show what it is. EL is the electric light placed in a lantern or box of some kind, to prevent the room, which should be slightly darkened, from being flooded with bright white light. L_1 is a condenser which

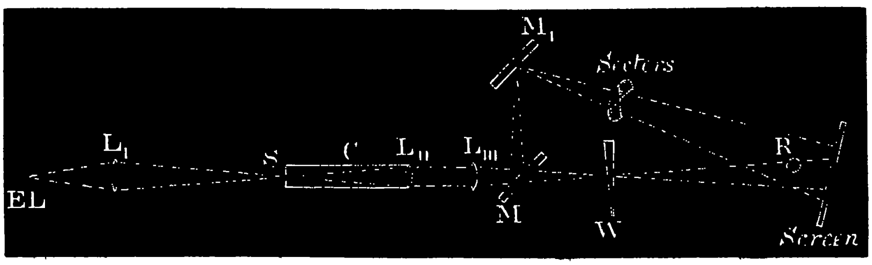

Fig. 1.

throws an image of the crater of the positive pole upon the slit S of the collimator C. The rays issue parallel and are caught by a lens L_{111}, which forms an image of the slit upon the surface of the wedge W, placed in a proper position and in its mountings. The light after passing through the wedge forms a circle of light on the screen. It will be noticed that the image of the slit may be as narrow as one wishes by opening or closing S, and that we have a line of light passing through the wedge, such as is required to effect the graduation. Calculation will show that with a fairly narrow slit the measured intensity passing through it may be taken as that passing through the mean thickness of that part on which the image falls.

Placed in the path of the beam, and between the wedge and L_{111}, is a plain mirror M (for which I often substitute a prism of $1\frac{1}{2}°$, and so obtain a single reflection), which reflects the light at right angles, or any convenient angle to its path. It is again